For each of the following functions, find the derivative and simplify.

1) $f(x)=\sqrt{1+(x-3)^2}$

Solution

$$f(x)=\left[1+(x-3)^2\right]^{\frac{1}{2}}$$
$$f'(x)=\frac{1}{2}\left[1+(x-3)^2\right]^{-\frac{1}{2}}(2)(x-3)^1(1)$$
$$=\frac{2(x-3)}{2\sqrt{1+(x-3)^2}}$$
$$=\frac{x-3}{\sqrt{1+(x-3)^2}}$$

2) $f(x)=(x^2+3)^4(4x-5)^3$

Solution

$$f'(x)=4(x^2+3)^3(2x)(4x-5)^3+3(4x-5)^2(4)(x^2+3)^4$$
$$=8x(x^2+3)^3(4x-5)^3+12(4x-5)^2(x^2+3)^4$$
$$=4(x^2+3)^3(4x-5)^2\left[2x(4x-5)+3(x^2+3)\right]$$
$$=4(x^2+3)^3(4x-5)^2\left(11x^2-10x+9\right)$$

3) $g(x)=\dfrac{2x}{\sqrt[3]{x^2+4}}$

Solution

$$g(x)=\frac{2x}{(x^2+4)^{\frac{1}{3}}}$$
$$g'(x)=\frac{2(x^2+4)^{\frac{1}{3}}-\frac{1}{3}(x^2+4)^{-\frac{2}{3}}(2x)(2x)}{\left[(x^2+4)^{\frac{1}{3}}\right]^2}$$
$$=\frac{2(x^2+4)^{\frac{1}{3}}-\frac{1}{3}(4x^2)(x^2+4)^{-\frac{2}{3}}}{(x^2+4)^{\frac{2}{3}}}$$
$$=\frac{\frac{1}{3}(x^2+4)^{-\frac{2}{3}}\left[6(x^2+4)-4x^2\right]}{(x^2+4)^{\frac{2}{3}}}$$
$$=\frac{2x^2+24}{3(x^2+4)^{\frac{2}{3}}(x^2+4)^{\frac{2}{3}}}$$
$$=\frac{2x^2+24}{3(x^2+4)^{\frac{4}{3}}}$$
$$=\frac{2(x^2+12)}{3\sqrt[3]{(x^2+4)^4}}$$

4) $g(x)=\left(\dfrac{1+x^2}{1-x^2}\right)^{10}$

Solution

$$g'(x)=10\left(\frac{1+x^2}{1-x^2}\right)^9\left[\frac{2x(1-x^2)-(-2x)(1+x^2)}{(1-x^2)^2}\right]$$
$$=10\left(\frac{1+x^2}{1-x^2}\right)^9\left[\frac{4x}{(1-x^2)^2}\right]$$
$$=\frac{40x(1+x^2)^9}{(1-x^2)^9(1-x^2)^2}$$
$$=\frac{40x(1+x^2)^9}{(1-x^2)^{11}}$$

*NOTE: The functions in examples 3 and 4 could also be differentiated using the product rule instead of the quotient rule.

5) $f(t)=\left(\frac{\sqrt[3]{5+3t}}{1-t^2}\right)^2$

Solution

$$f(t)=\left[\frac{(5+3t)^{\frac{1}{3}}}{1-t^2}\right]^2$$

$$=\frac{(5+3t)^{\frac{2}{3}}}{\left(1-t^2\right)^2}$$

$$f'(t)=\frac{\frac{2}{3}(5+3t)^{-\frac{1}{3}}(3)\left(1-t^2\right)^2-2\left(1-t^2\right)(-2t)(5+3t)^{\frac{2}{3}}}{\left[\left(1-t^2\right)^2\right]^2}$$

$$=\frac{2(5+3t)^{-\frac{1}{3}}\left(1-t^2\right)^2+4t\left(1-t^2\right)(5+3t)^{\frac{2}{3}}}{\left(1-t^2\right)^4}$$

$$=\frac{2(5+3t)^{-\frac{1}{3}}\left(1-t^2\right)\left[\left(1-t^2\right)+2t(5+3t)\right]}{\left(1-t^2\right)^4}$$

$$=\frac{2\left(5t^2+10t+1\right)}{(5+3t)^{\frac{1}{3}}\left(1-t^2\right)^3}$$

$$=\frac{2\left(5t^2+10t+1\right)}{\sqrt[3]{5+3t}\left(1-t^2\right)^3}$$

*NOTE: This function could also be differentiated using the product rule instead of the quotient rule.

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6) $f(x)=\frac{(x^2+3)^3(x^3-1)^2}{x^4}$

Solution

$$f'(x)=\frac{\left[3\left(x^2+3\right)^2(2x)\left(x^3-1\right)^2+2\left(x^3-1\right)^1(3x^2)\left(x^2+3\right)^3\right]x^4-4x^3\left(x^2+3\right)^3\left(x^3-1\right)^2}{\left(x^4\right)^2}$$

$$=\frac{6x^5\left(x^2+3\right)^2\left(x^3-1\right)^2+6x^6\left(x^3-1\right)^1\left(x^2+3\right)^3-4x^3\left(x^2+3\right)^3\left(x^3-1\right)^2}{x^8}$$

$$=\frac{2x^3\left(x^2+3\right)^2\left(x^3-1\right)\left[3x^2\left(x^3-1\right)+3x^3\left(x^2+3\right)-2\left(x^2+3\right)\left(x^3-1\right)\right]}{x^8}$$

$$=\frac{2\left(x^2+3\right)^2\left(x^3-1\right)\left(3x^5-3x^2+3x^5+9x^3-2x^5+2x^2-6x^3+6\right)}{x^5}$$

$$=\frac{2\left(x^2+3\right)^2\left(x^3-1\right)\left(4x^5+3x^3-x^2+6\right)}{x^5}$$