The Cartesian Equation of a Line in R^2

The equation of a line written in the form $Ax + By + C = 0$ is called the **Cartesian equation** or **scalar equation** of the line.

Example
Write the Cartesian equation of the line $\vec{r} = (5,4) + t(-3,8)$.

The slope of the line is $-\frac{8}{3}$

$\therefore y = -\frac{8}{3}x + b$

Since (5, 4) is a point on the line,

$4 = -\frac{8}{3}(5) + b$

$b = \frac{52}{3}$

$\therefore y = -\frac{8}{3}x + \frac{52}{3}$

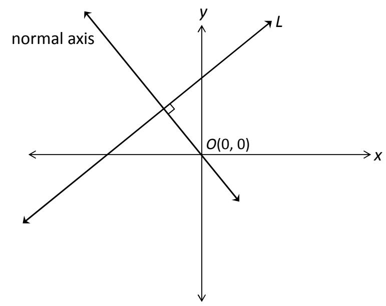
$3y = -8x + 52$

$\boxed{8x + 3y - 52 = 0}$

The Normal Axis and Normals

To develop a deeper understanding of Cartesian equations of lines, we need to become familiar with the ideas of **normals** and the **normal axis**.

What is a normal axis?
For a line L in R^2, the normal axis is the line that is perpendicular to L and that passes through the origin.

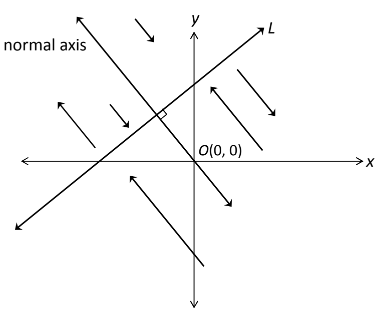


Is the normal axis of a line unique?

YES!

There is only one line that is perpendicular to *L* and that also passes through the origin.

Normal Vectors



What is a normal vector?
A **normal vector** to the line *L* is any non-zero vector that is parallel to L's normal axis.

In other words, a normal vector to line *L* is any non-zero vector that is perpendicular to *L*.

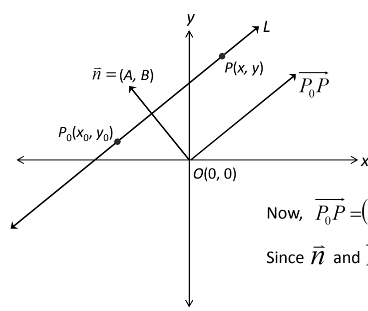
The diagram on the left shows several normal vectors for the line *L*.

Notes about normal vectors:

- normal vectors are often referred to simply as "normals"
- a normal to a line is perpendicular to any vector on the line

Connecting Normals to Cartesian Equations

To see how a normal of a line is related to its Cartesian (scalar) equation, consider the following demonstration:



Let L be a line in R^2.

Let $P(x, y)$ represent any point on L.

Let $P_0(x_0, y_0)$ represent a specific point on L, with known coordinates.

Let $\vec{n} = (A, B)$ be a normal to L.

Now, $\overrightarrow{P_0P} = (x - x_0, y - y_0)$

Since $\vec{n}$ and $\overrightarrow{P_0P}$ are perpendicular, we can write

$$\vec{n} \cdot \overrightarrow{P_0P} = 0$$
$$(A, B) \cdot (x - x_0, y - y_0) = 0$$
$$Ax - Ax_0 + By - By_0 = 0$$
$$Ax + By - Ax_0 - By_0 = 0$$

$$Ax + By - Ax_0 - By_0 = 0$$

We can represent the known constant $-Ax_0 - By_0$ with *C*.

$$Ax + By + C = 0$$

Do you recognize the above expression?

It's the Cartesian equation of the line!

Conclusion

The coefficients of *x* and *y* in a line's Cartesian equation are the components of one of its normal vectors.

LET'S DO SOME EXAMPLES!

Examples (complete on a separate page)

1) State two normals to the line $6x - 5y + 4 = 0$.

2) Determine the Cartesian equation of the line passing through $A(4, -2)$, which has $\vec{n} = (5,3)$ as a normal.

3) Show that the lines $L_1 : 3x - 4y - 6 = 0$ and $L_2 : 6x - 8y + 12 = 0$ are parallel and non-coincident.

4) For what value of *k* are the lines $L_1 : kx + 4y - 4 = 0$ and $L_2 : 3x - 2y - 3 = 0$ perpendicular?

5) Determine the acute angle at the point of intersection of the lines $L_1 : 2x + 3y - 1 = 0$ and $L_2 : 4x - 3y + 6 = 0$.