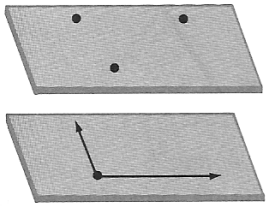
# EQUATION OF A PLANE

## Equation of a Plane



What information do we need in order to determine a plane?

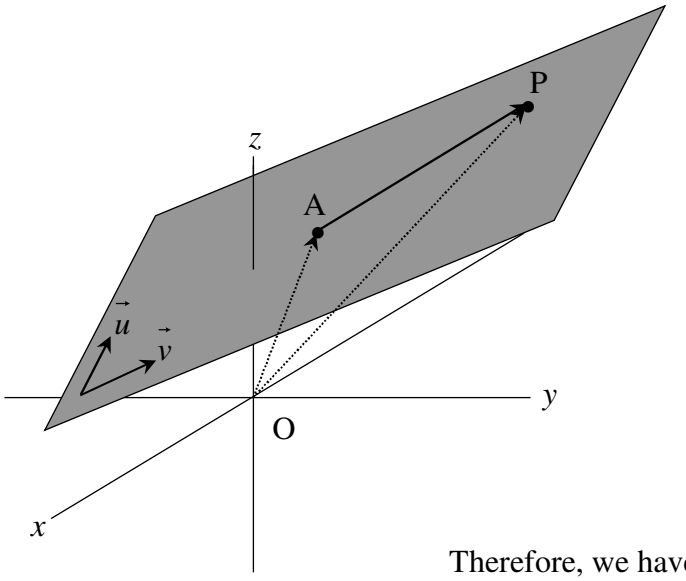
1) _____

or 2) _____

Using two non-collinear direction vectors to specify the "slant" of a plane and one point on the plane to fix its location in space, we can develop a **vector equation** of the plane.

## Vector Equation of a Plane

Consider the following diagram, in which A($a_1$, $a_2$, $a_3$) is a known point on the given plane and P($x$, $y$, $z$) is any point on the plane. Also, let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ be two non-collinear direction vectors of the plane.




Notice that regardless of the location of point P on the plane, we always have $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$.

Furthermore, since P is a point on the plane, the vector $\overrightarrow{AP}$ can be written as a linear combination of direction vectors $\vec{u}$ and $\vec{v}$.

Therefore, for scalars $s$ and $t$, $\overrightarrow{AP} =$

Also, $\overrightarrow{OP} =$ and $\overrightarrow{OA} =$

Therefore, we have _____.

**Vector Equation of a Plane**

Let A($a_1$, $a_2$, $a_3$) be a fixed point on a plane with direction vectors $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$. Let P($x$, $y$, $z$) be any point on the plane. A **vector equation** of the plane is

$$(x, y, z) = (a_1, a_2, a_3) + s(u_1, u_2, u_3) + t(v_1, v_2, v_3)$$

where $s$ and $t$ are any scalars.

Is the vector equation of a plane unique?

Why are there two direction vectors in the equation of a plane?

## Parametric Equations of a Plane

By simplifying the right-hand side of the vector equation $(x, y, z) = (a_1, a_2, a_3) + s(u_1, u_2, u_3) + t(v_1, v_2, v_3)$ and equating the corresponding components of the equal vectors, we can develop equations for $x$, $y$ and $z$ in terms of parameters $s$ and $t$. These three equations are called **parametric equations** of the plane.

**Parametric Equations of a Plane**

Let A$(a_1, a_2, a_3)$ be a fixed point on a plane with direction vectors $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$. Let P$(x, y, z)$ be any point on the plane. **Parametric equations** of the plane are

$$x = a_1 + su_1 + tv_1$$
$$y = a_2 + su_2 + tv_2$$
$$z = a_3 + su_3 + tv_3$$

where $s$ and $t$ are any scalars.

**Example**
Find vector and parametric equations of the plane through the points A(1, 7, 2), B(4, 0, -1) and C(1, 2, 3). Find two other points on the plane.