EQUATION OF A LINE IN 3-SPACE

Equation of a Line in 3-Space

The results that we developed for the equation of a line in 2-space can also be extended to 3-space.

Vector Equation of a Line in 3-Space

Let A(a_1, a_2, a_3) be a fixed point on a line in 3-space with a direction vector $\vec{d} = (d_1, d_2, d_3)$ and let P(x, y, z) be any point on the line. A **vector equation** of the line is

$$(x, y, z) = (a_1, a_2, a_3) + t(d_1, d_2, d_3)$$

where t is any scalar.

Parametric Equations of a Line in 3-Space

By simplifying the right-hand side of the vector equation $(x, y, z) = (a_1, a_2, a_3) + t(d_1, d_2, d_3)$ and equating the corresponding components of the equal vectors, we can develop equations for x, y and z in terms of t. These three equations are called **parametric equations** of the line.

Parametric Equations of a Line in 3-Space

Let A(a_1, a_2, a_3) be a fixed point on a line in 3-space with a direction vector $\vec{d} = (d_1, d_2, d_3)$ and let P(x, y, z) be any point on the line. **Parametric equations** of the line are

$$\begin{aligned} x &= a_1 + td_1 \\ y &= a_2 + td_2 \\ z &= a_3 + td_3 \end{aligned}$$

where t is any scalar.

Symmetric Equations of a Line in 3-Space

If we isolate t in each of the parametric equations we obtain the following.

The above equations are called **symmetric equations** of the line. Notice that there are actually two equations in the above expression.

Symmetric Equations of a Line in 3-Space

Let A(a_1, a_2, a_3) be a fixed point on a line in 3-space with a direction vector $\vec{d} = (d_1, d_2, d_3)$ and let P(x, y, z) be any point on the line. **Symmetric equations** of the line are

$$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$$

where $d_1 \neq 0$, $d_2 \neq 0$ and $d_3 \neq 0$.

Example
Find vector, parametric and symmetric equations for the line through the points P(-2, 3, 5) and Q(-2, 4, -1).

Example
Show that $L_1 : \vec{r} = (-1, 0, 4) + k(-1, 2, 5)$ and $L_2 : \vec{r} = (4, -10, -21) + l(-2, 4, 10)$ are equations for the same line.