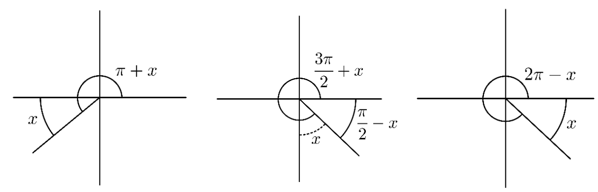
1) Express $\dfrac{2\tan\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)}$ as a **single** trigonometric function. Simplify as much as possible.

$$\frac{2\tan\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)} = \tan\left[2\left(\frac{\theta}{2}\right)\right]$$
$$= \tan\theta$$

2) Simplify $\sin\left(\frac{\pi}{2}-x\right)+\sin(\pi+x)+\sin\left(\frac{3\pi}{2}+x\right)+\sin(2\pi-x)$.

$$\sin\left(\frac{\pi}{2}-x\right)+\sin(\pi+x)+\sin\left(\frac{3\pi}{2}+x\right)+\sin(2\pi-x)$$
$$=\cos x-\sin x-\sin\left(\frac{\pi}{2}-x\right)-\sin x$$
$$=\cos x-\sin x-\cos x-\sin x$$
$$=-2\sin x$$

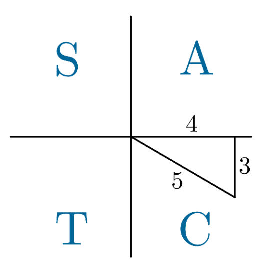



3) Determine the exact value of the trigonometric ratio $\sin\frac{5\pi}{12}$. Rationalize the denominator.

$$\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)$$
$$=\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$$
$$=\sin\frac{\pi}{6}\cos\frac{\pi}{4}+\cos\frac{\pi}{6}\sin\frac{\pi}{4}$$
$$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$=\frac{1}{2\sqrt{2}}+\frac{\sqrt{3}}{2\sqrt{2}}$$
$$=\frac{1+\sqrt{3}}{2\sqrt{2}}\;\frac{\times\sqrt{2}}{\times\sqrt{2}}$$
$$=\frac{\sqrt{2}+\sqrt{6}}{4}$$

4) If $\tan x=-\frac{3}{4}$ and $\frac{3\pi}{2}\le x\le 2\pi$, determine the exact value of $\sin 4x$.

$$\sin 4x=\sin\left[2(2x)\right]$$
$$=2\sin(2x)\cos(2x)$$
$$=2(2\sin x\cos x)(2\cos^2 x-1)$$
$$=2\left[2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)\right]\left[2\left(\frac{4}{5}\right)^2-1\right]$$
$$=-\frac{336}{625}$$



5) Determine the solutions for each equation on the interval $0 \le x \le 2\pi$. Give exact solutions, where possible. Round approximate solutions to the nearest hundredth of a radian.

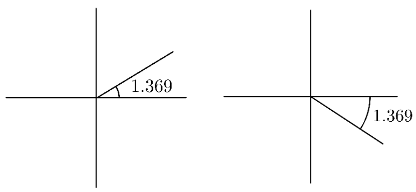
a) $-5\cos x + 3 = 2$

$$-5\cos x + 3 = 2$$

$$\cos x = \frac{1}{5}$$

R.A.A. = 1.369

Quadrants 1 and 4




$x \approx 1.37$ $\qquad$ $x = 2\pi - 1.369$

$\approx 4.91$

$\therefore x = 1.37, 4.91$

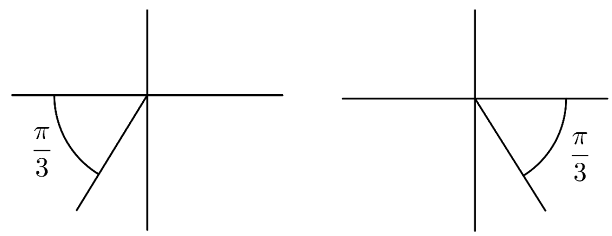
b) $2\sin 3x + \sqrt{3} = 0$ $\qquad$ $\boxed{\begin{array}{c} 0 \le x \le 2\pi \\ 0 \le 3x \le 6\pi \end{array}}$

$$\sin 3x = -\frac{\sqrt{3}}{2}$$

<u>For $3x$</u>

R.A.A. $= \frac{\pi}{3}$

Quadrants 3 and 4




$3x = \frac{4\pi}{3}, \frac{10\pi}{3}, \frac{16\pi}{3}$ $\qquad$ $3x = \frac{5\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}$

$x = \frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}$ $\qquad$ $x = \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$

$$\therefore x = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$$

c) $\sin x + \sin x \tan x = 0$

$$\sin x(1 + \tan x) = 0$$

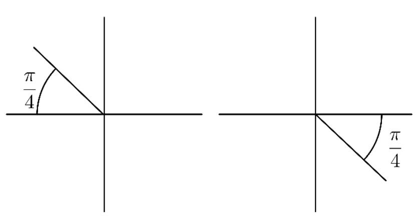
$\sin x = 0$ $\qquad$ $1 + \tan x = 0$

$\tan x = -1$

R.A.A. $= \frac{\pi}{4}$

Quadrants 2 and 4

$x = 0, \pi, 2\pi$




$x = \frac{3\pi}{4}$ $\qquad$ $x = \frac{7\pi}{4}$

$$\therefore x = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$$

d) $\cos 2x - 3 = 5\cos x - 4\cos^2 x$

$$\left(2\cos^2 x - 1\right) - 3 = 5\cos x - 4\cos^2 x$$

$$6\cos^2 x - 5\cos x - 4 = 0$$

$$(2\cos x + 1)(3\cos x - 4) = 0$$

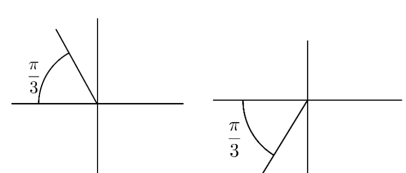
$2\cos x + 1 = 0$ $\qquad$ $3\cos x - 4 = 0$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = \frac{4}{3}$

No solution

R.A.A. $= \frac{\pi}{3}$

Quadrants 2 and 3




$x = \frac{2\pi}{3}$ $\qquad$ $x = \frac{4\pi}{3}$

$$\therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

6) Prove the following identities.

a) $\dfrac{\sin 2x+\sin x}{1+\cos x+\cos 2x}=\cot\left(\dfrac{\pi}{2}-x\right)$

Left Side

$$\frac{\sin 2x+\sin x}{1+\cos x+\cos 2x}$$
$$=\frac{2\sin x\cos x+\sin x}{1+\cos x+2\cos^2 x-1}$$
$$=\frac{2\sin x\cos x+\sin x}{\cos x+2\cos^2 x}$$
$$=\frac{\sin x(2\cos x+1)}{\cos x(1+2\cos x)}$$
$$=\frac{\sin x}{\cos x}$$

Right Side

$$\cot\left(\frac{\pi}{2}-x\right)$$
$$=\frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$$
$$=\frac{\sin x}{\cos x}$$

∴ left side = right side
∴ identity proven

b) $(\sec x-\cos x)(\csc x-\sin x)=\dfrac{\tan x}{1+\tan^2 x}$

Left Side

$$(\sec x-\cos x)(\csc x-\sin x)$$
$$=\left(\frac{1}{\cos x}-\cos x\right)\left(\frac{1}{\sin x}-\sin x\right)$$
$$=\frac{1}{\cos x\sin x}-\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}+\cos x\sin x$$
$$=\frac{1-\sin^2 x-\cos^2 x+\cos^2 x\sin^2 x}{\cos x\sin x}$$
$$=\frac{1-\left(\sin^2 x+\cos^2 x\right)+\cos^2 x\sin^2 x}{\cos x\sin x}$$
$$=\frac{1-1+\cos^2 x\sin^2 x}{\cos x\sin x}$$
$$=\frac{\cos^2 x\sin^2 x}{\cos x\sin x}$$
$$=\cos x\sin x$$

Right Side

$$\frac{\tan x}{1+\tan^2 x}$$
$$=\frac{\frac{\sin x}{\cos x}}{\sec^2 x}$$
$$=\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$$
$$=\frac{\sin x}{\cos x}\times\frac{\cos^2 x}{1}$$
$$=\sin x\cos x$$
$$=\cos x\sin x$$

∴ left side = right side
∴ identity proven

7) Determine the exact value of $\tan\frac{\pi}{8}$. **Show all work and simplify your answer as much as possible**.

$$\tan\left[2\left(\frac{\pi}{8}\right)\right]=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$

$$\tan\frac{\pi}{4}=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$

$$1=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$

$$1-\tan^2\frac{\pi}{8}=2\tan\frac{\pi}{8}$$

$$0=\tan^2\frac{\pi}{8}+2\tan\frac{\pi}{8}-1$$

$$\tan\frac{\pi}{8}=\frac{-2\pm\sqrt{2^2-4(1)(-1)}}{2(1)}$$
$$=\frac{-2\pm\sqrt{8}}{2}$$
$$=\frac{-2\pm2\sqrt{2}}{2}$$
$$=-1\pm\sqrt{2}$$

Since $\frac{\pi}{8}$ is in quadrant 1, $\tan\frac{\pi}{8}$ is positive.

$$\therefore \tan\frac{\pi}{8}=-1+\sqrt{2}$$

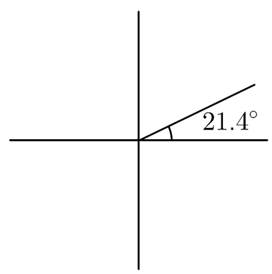
8) Solve $\csc\theta=2.7451$ for $-720°\le\theta\le720°$. Round your answers to the nearest tenth of a degree.

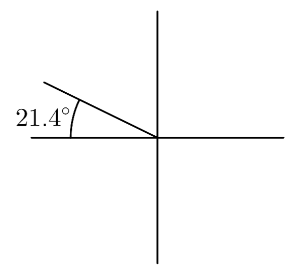
$\csc\theta=2.7451$

$\sin\theta=\frac{1}{2.7451}$

R.A.A. $\approx 21.4°$

Quadrants 1 and 2




$\theta=21.4°, 381.4°, -338.6°, -698.6°$ $\qquad$ $\theta=158.6°, 518.6°, -201.4°, -561.4°$

$$\therefore \theta=-698.6°, -561.4°, -338.6°, -201.4°, 21.4°, 158.6°, 381.4°, 518.6°$$

9) Using other identities, develop the tangent subtraction formula $\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$.

$$\tan(x-y)=\tan\left[x+(-y)\right]$$
$$=\frac{\tan x+\tan(-y)}{1-\tan x\tan(-y)}$$
$$=\frac{\tan x+(-\tan y)}{1-\tan x(-\tan y)}$$
$$=\frac{\tan x-\tan y}{1+\tan x\tan y}$$