A circle with centre $(0,0)$ has the equation $x^2 + y^2 = 100$.

a) Determine the radius of the circle.

b) The points $A(0, \_)$, $B(\_, 0)$ and $C(\_, \_)$ are on the circle. Determine a possible value for each blank. The point C must not contain any zeros in its coordinates. Provide calculation to prove that the points lie on the circle.

c) Determine the equations of the perpendicular bisectors of the chords AB and AC.

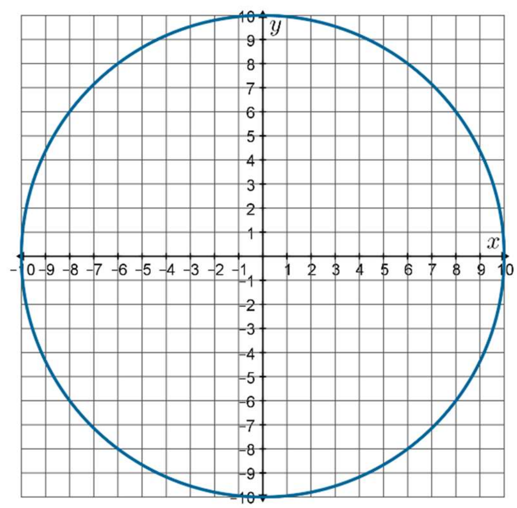
d) Show that the perpendicular bisectors from part (c) intersect at the point (0, 0).

a) The equation of a circle centred at the origin is $x^2 + y^2 = r^2$, where r is the radius of the circle. For the given equation, we have

$$x^2 + y^2 = 100$$
$$x^2 + y^2 = 10^2$$

∴the radius of the circle is 10.

b) A graph of the circle with centre (0, 0) and radius 10 is shown on the right. Based on the graph, we see that (0, 10) and (10, 0) are points on the circle. It also appears that the point (8, 6) is on the circle. We can confirm algebraically by checking that these points satisfy the circle's equation (check that the left and right sides are equal).



For the point (0, 10):

Left Side	Right Side
$x^2 + y^2$	100
$= x^2 + y^2$	
$= (0)^2 + (10)^2$	∴ left side = right side
$= 100$	∴(0, 10) is on the circle

For the point (10, 0):

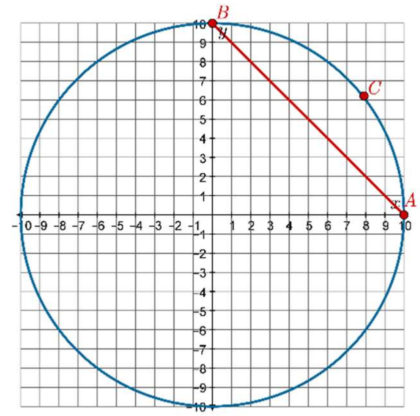
Left Side	Right Side
$x^2 + y^2$	100
$= x^2 + y^2$	
$= (10)^2 + (0)^2$	∴ left side = right side
$= 100$	∴(10, 0) is on the circle

For the point (8, 6):

Left Side	Right Side
$x^2 + y^2$	100
$= x^2 + y^2$	
$= (8)^2 + (6)^2$	∴ left side = right side
$= 100$	∴(8, 6) is on the circle

∴ possible points are $A(0, 10)$, $B(10, 0)$ and $C(8, 6)$. *NOTE: Other correct answers are possible.*

c) Chord *AB* is shown on the right.



Find the midpoint of *AB*:

$$\text{Midpoint}_{AB} = \left(\frac{10+0}{2}, \frac{0+10}{2}\right)$$
$$= (5, 5)$$

Find the slope of *AB*:

$$\text{Slope}_{AB} = \frac{10-0}{0-10}$$
$$= -1$$

∴ the perpendicular slope is

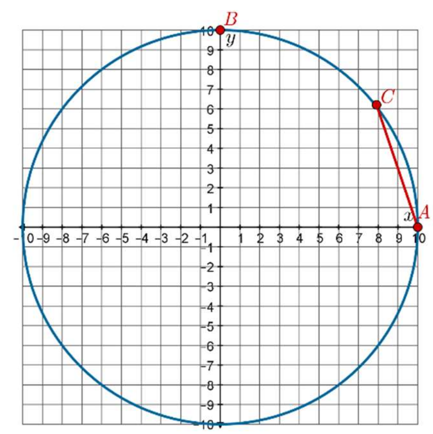
$$-\frac{1}{-1}$$
$$= 1$$

So, the perpendicular bisector has a slope of 1 and passes through the point (5, 5). Therefore,

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 1(x - 5)$$
$$y - 5 = x - 5$$
$$y = x - 5 + 5$$
$$y = x$$

∴ the equation of the perpendicular bisector of *AB* is $y = x$.

Chord *AC* is shown on the right.



Find the midpoint of *AC*:

$$\text{Midpoint}_{AC} = \left(\frac{10+8}{2}, \frac{0+6}{2}\right)$$
$$= (9, 3)$$

Find the slope of *AC*:

$$\text{Slope}_{AC} = \frac{6-0}{8-10}$$
$$= -3$$

∴ the perpendicular slope is

$$-\frac{1}{-3}$$
$$= \frac{1}{3}$$

So, the perpendicular bisector has a slope of $\frac{1}{3}$ and passes through the point (9, 3). Therefore,

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{1}{3}(x - 9)$$
$$y - 3 = \frac{1}{3}x - 3$$
$$y = \frac{1}{3}x - 3 + 3$$
$$y = \frac{1}{3}x$$

∴ the equation of the perpendicular bisector of *AC* is $y = \frac{1}{3}x$.

c) Use the method of substitution (or elimination) to find where the lines $y = x$ and $y = \frac{1}{3}x$ intersect.

① $y = x$

② $y = \frac{1}{3}x$

Substitute ① into ②: $y = \frac{1}{3}x$

$$(x) = \frac{1}{3}x$$

multiply by 3

$$3x = x$$

$$3x - x = 0$$

$$2x = 0$$

$$x = \frac{0}{2}$$

$$x = 0$$

Substitute $x = 0$ into ① (or ②): $y = x$

$$y = (0)$$

$$y = 0$$

∴ the perpendicular bisectors intersect at the point (0, 0).