Let $g(x) = x^2\sqrt{f(x)}$, where f is a differentiable function such that $f(1) = 4$ and $f'(1) = 1$. Find $g'(1)$.

$$g(x) = x^2\sqrt{f(x)}$$
$$= x^2\left[f(x)\right]^{\frac{1}{2}}$$
$$g'(x) = 2x\left[f(x)\right]^{\frac{1}{2}} + \frac{1}{2}\left[f(x)\right]^{-\frac{1}{2}} f'(x)x^2$$
$$= 2x\sqrt{f(x)} + \frac{1}{2}\left(\frac{1}{\sqrt{f(x)}}\right)f'(x)x^2$$

Product rule (*derivative of first times second plus derivative of second times first*). Notice the use of the chain rule when taking the derivative of the "second function." Since we don't know what the "inner" function $f(x)$ is, we just write $f'(x)$ for its derivative.

$$g'(1) = 2(1)\sqrt{f(1)} + \frac{1}{2}\left(\frac{1}{\sqrt{f(1)}}\right)f'(1)(1)^2$$
$$= 2(1)\sqrt{4} + \frac{1}{2}\left(\frac{1}{\sqrt{4}}\right)(1)(1)^2$$
$$= 4 + \frac{1}{4}$$
$$= \frac{17}{4}$$

Substitute values given in question.